Exercise 54

(a) Evaluate $\lim_{x\to\infty} x\sin\dfrac{1}{x}$.

(b) Evaluate $\lim_{x\to 0} x\sin\dfrac{1}{x}$.

(c) Illustrate parts (a) and (b) by graphing $y = x\sin(1/x)$.

Solution

Rewrite the limits in terms of one that is known.

$$\lim_{x\to\infty} x\sin\frac{1}{x} = \lim_{x\to\infty} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1$$

$$\lim_{x\to 0} x\sin\frac{1}{x} = \lim_{x\to 0} \frac{\sin\frac{1}{x}}{\frac{1}{x}} = \lim_{\theta\to\infty}\frac{\sin\theta}{\theta} = 0$$

These limits are illustrated in the plot below of $y = x\sin(1/x)$ versus x.